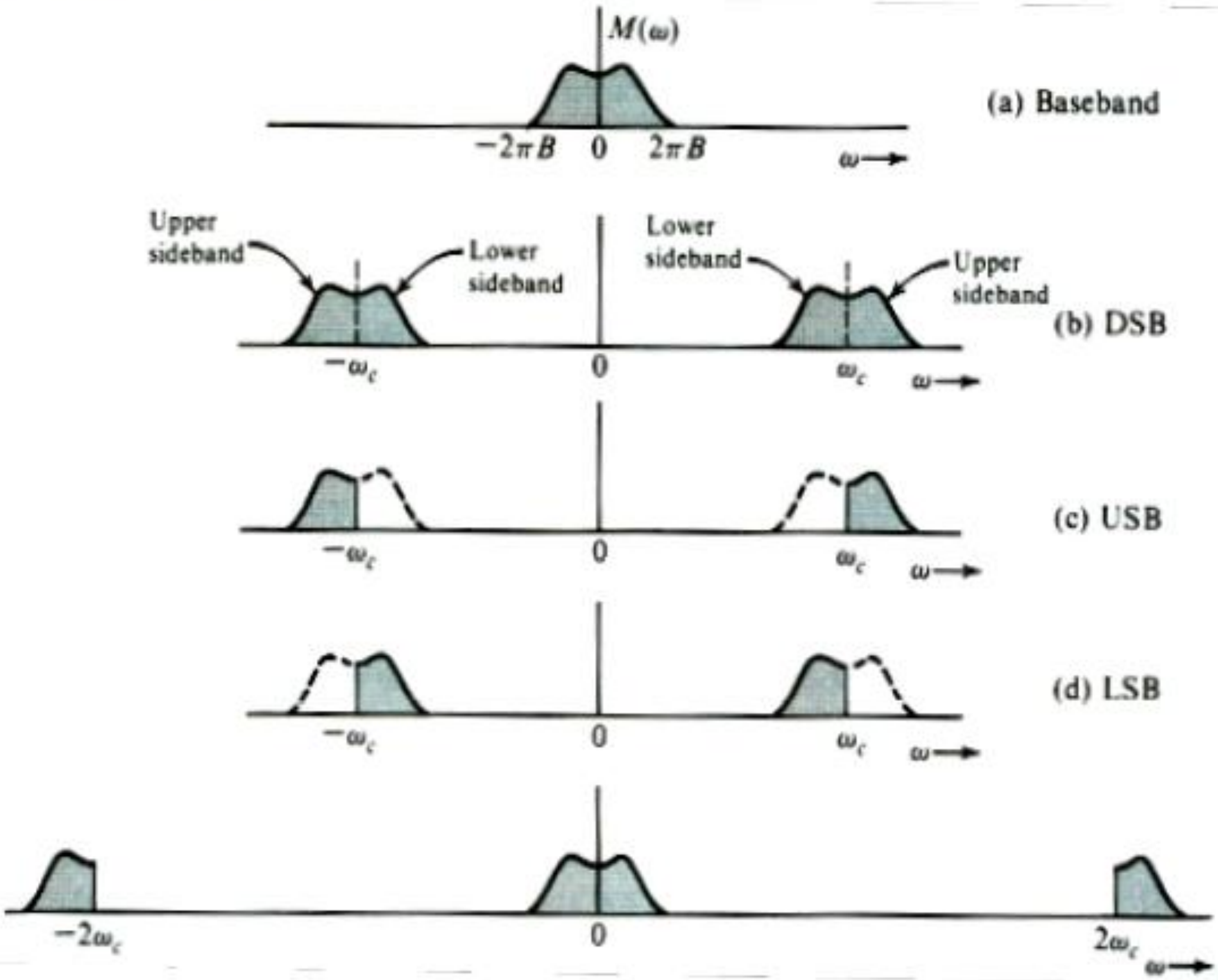
Communication Systems

Lecture 10

Amplitude Modulation: Single Sideband

- Amplitude Modulation Single Side Band Also known as SSB-SC
- A scheme in which only one sideband (USB or LSB) is transmitted
- Requires only one half the bandwidth of the DSB signal
- Demodulation of SSB signals is identical to that of DSB-SC signals

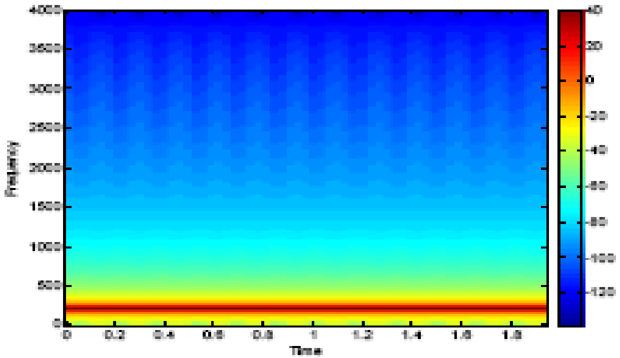
SSB Spectra



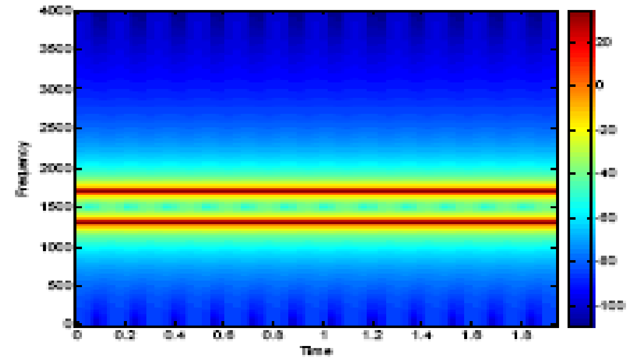
$M(\omega)$
(a) Baseband
$-2\pi B$
0
$2\pi B$
$\omega \rightarrow$
Upper sideband
Lower sideband
Lower sideband
Upper sideband
(b) DSB
$-\omega_c$
0
ω_c
$\omega \rightarrow$
(c) USB
$-\omega_c$
0
ω_c
$\omega \rightarrow$
(d) LSB
$-\omega_c$
0
ω_c
$\omega \rightarrow$
$-2\omega_c$
0
$2\omega_c$
$\omega \rightarrow$

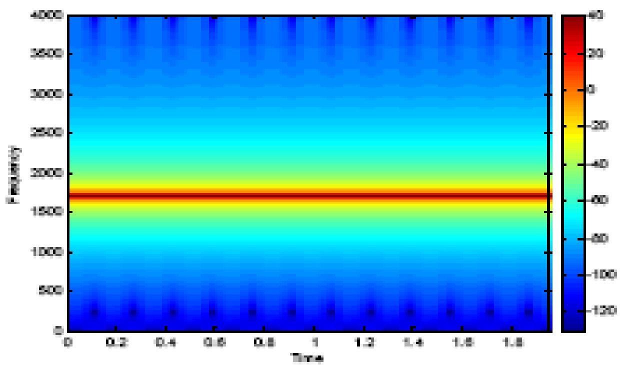
Motivation
Tone Modulation: Simple Sinusoid Case

100 Hz

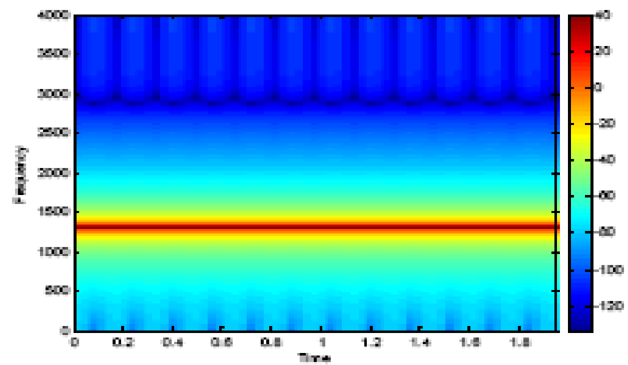


DSB-SC, 1500 Hz Carrier





SSB-USB



SSB-LSB

Time Domain Representation of SSB Signals

1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\text{sgn}\, t$	$\frac{2}{j\omega}$

$$M_+(\omega) = M(\omega)U(\omega) = M(\omega)\left[\frac{1+\text{sgn}(\omega)}{2}\right] = \tfrac{1}{2}\left[M(\omega) + M(\omega)\text{sgn}(\omega)\right] \quad \Leftrightarrow \quad m_+(t) = \tfrac{1}{2}\text{m(t)} + \tfrac{1}{2}\mathcal{F}^{-1}\{M(\omega)\} * \mathcal{F}^{-1}\{\text{sgn}(\omega)\}$$

$$\mathcal{F}^{-1}\{\text{sgn}(\omega)\} = -\frac{1}{j\pi t} = \frac{j}{\pi t} \qquad \therefore \quad m_+(t) = \tfrac{1}{2}\left(\text{m(t)} + jm(t) * \frac{1}{\pi t}\right) = \tfrac{1}{2}\left(\text{m(t)} + jm_h(t)\right)$$

$$\text{where } m_h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(\lambda)}{t-\lambda}d\lambda$$

$m_h(t)$ is called the Hilbert transform of m(t).

$$\text{Similarly, we can show that } m_-(t) = \tfrac{1}{2}\left(\text{m(t)} - jm(t) * \frac{1}{\pi t}\right) = \tfrac{1}{2}\left(\text{m(t)} - jm_h(t)\right)$$

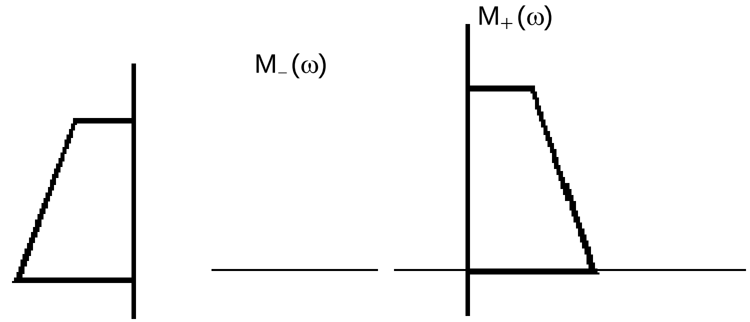
$$\mathbf{M_H(\omega)} = -j\,\mathbf{M(\omega)\ sgn(\omega)}$$

Time Domain Representation of SSB Signals

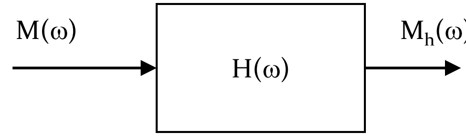
$$M_H(\omega) = -jM(\omega)\,\mathrm{sgn}(\omega)$$

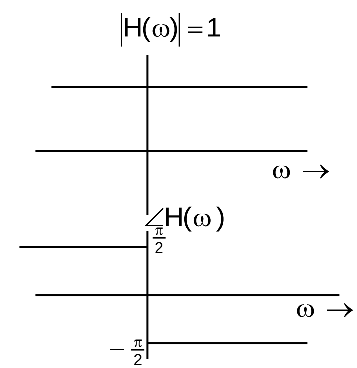
$$H(\omega) = \frac{M_H(\omega)}{M(\omega)} = -j\,\mathrm{sgn}(\omega)$$

$$= \begin{cases} -j & \text{for } \omega > 0 \\ j & \text{for } \omega < 0 \end{cases}$$



Transfer function of a Hilbert transformer



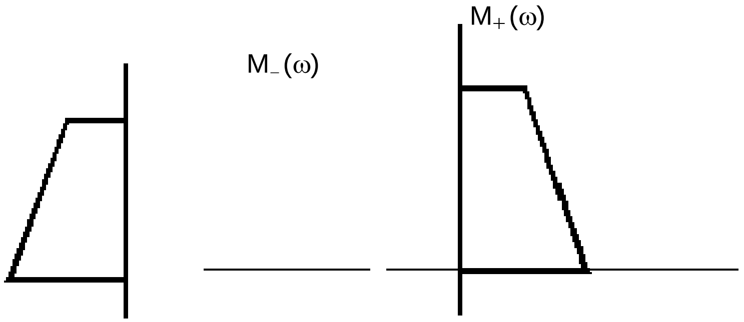


Time Domain Representation of SSB Signals

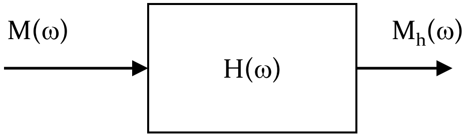
$$M_H(\omega) = -jM(\omega)\,\mathrm{sgn}(\omega)$$

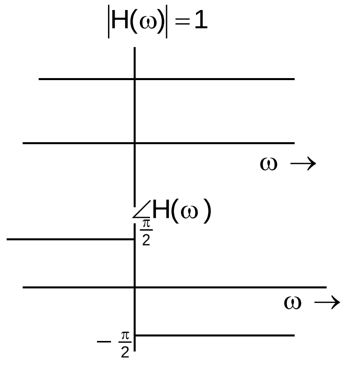
$$H(\omega) = \frac{M_H(\omega)}{M(\omega)} = -j\,\mathrm{sgn}(\omega)$$

$$= \begin{cases} -j & \text{for } \omega > 0 \\ j & \text{for } \omega < 0 \end{cases}$$



<u>Transfer function of a Hilbert transformer</u>





Time Domain Representation of SSB Signals

- SSB signal can be expressed in terms of m(t) and its Hilbert transform

$$\Phi_{SSB-USB}(\omega) = \left(M_+(\omega-\omega_c) + M_-(\omega+\omega_c)\right)$$

$$M_+(\omega-\omega_c) = \mathcal{F}^{-1}\{m_+(t)e^{j\omega_c t}\} = \mathcal{F}^{-1}\{\tfrac{1}{2}(m(t) + jm_h(t))e^{j\omega_c t}\}$$

$$M_-(\omega+\omega_c) = \mathcal{F}^{-1}\{m_-(t)e^{-j\omega_c t}\} = \mathcal{F}^{-1}\{\tfrac{1}{2}(m(t) - jm_h(t))e^{-j\omega_c t}$$

$$\therefore M_+(\omega-\omega_c) + M_-(\omega+\omega_c) = \mathcal{F}^{-1}\{\tfrac{1}{2}(m(t) + jm_h(t))e^{j\omega_c t} + \tfrac{1}{2}(m(t) - jm_h(t))e^{-j\omega_c t}\}$$

$$= \mathcal{F}^{-1}\left\{\tfrac{1}{2}m(t)(e^{j\omega_c t} + e^{j\omega_c t}) + j\tfrac{1}{2}m_h(t)(e^{j\omega_c t} - e^{-j\omega_c t})\right\}$$

$$= \mathcal{F}^{-1}\left\{m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)\right\}$$

$$\therefore \quad \Phi_{SSB-USB}(\omega) = \mathcal{F}^{-1}\left\{m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)\right\}$$

Similarly, we can show that $\Phi_{SSB-LSB}(\omega) = \mathcal{F}^{-1}\left\{m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)\right\}$;

In general, $\Phi_{SSB}(\omega) = \mathcal{F}^{-1}\left\{m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)\right\}$ (- for USB, + for LSB)

Time Domain Representation of SSB Signals

$$\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

$$\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

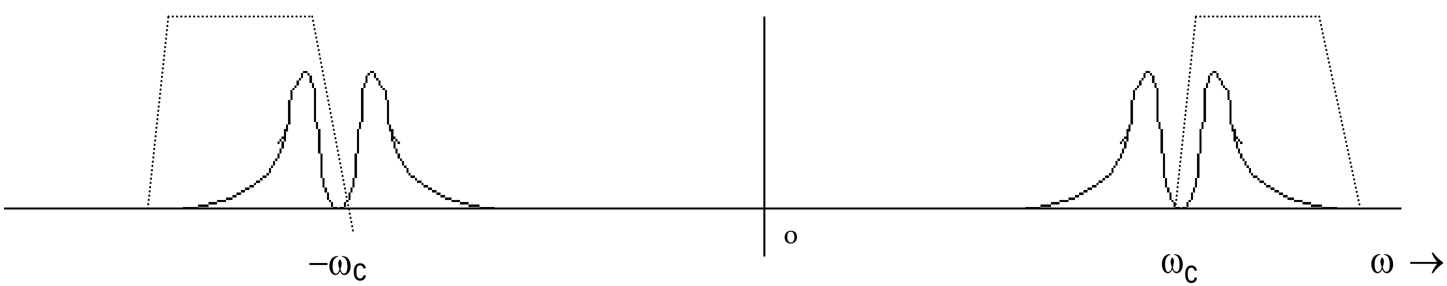
$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

Generation of SSB

- Selective filtering Method
  - A DSB-SC signal is passed through a sharp cuttoff filter to eliminate the undesired sideband
  - The filter should pass all components above w_c untenanted and completely suppress all components below w_c
  - Such an operation require an ideal filter, which is unrealizable
  - It can be realized closely if there is some separation between the passband and stopband e.g. Speech Signal (For speech frequencies below 300 Hz can be removed without effecting the quality)

SSB Generator

- Selective Filtering using filters with sharp cutoff characteristics. Sharp cutoff filters are difficult to design. The audio signal spectrum has no dc component, therefore , the spectrum of the modulated audio signal has a null around the carrier frequency. This means a less than perfect filter can do a reasonably good job of filtering the DSB to produce SSB signals.
- Baseband signal must be bandpass
- Filter design challenges
- No low frequency components



SSB Demodulation

Synchronous, SSB-SC demodulation

$$\varphi_{SSB}(t)\cos(\omega_c t) = \left[m(t)\cos(\omega_c t) \mp jm_h(t)\sin(\omega_c t)\right]\cos(n(\omega_c t) = \frac{1}{2}\left[m(t)(1+\cos(\omega_c t)) \mp jm_h(t)\sin(2\omega_c t)\right]$$

A lowpass filter can be used to get $\frac{1}{2}m(t)$.

SSB+C, envelop detection

$$\varphi_{SSB+C}(t) = A\cos(\omega_c t) + \left[m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)\right]$$

An envelope detector can be used to demodulate such SSB signals.

What is the envelope of $\varphi_{SSB+C}(t) = (A + m(t))\cos(\omega_c t)) + m_h(t)\sin(\omega_c t) = E(t)\cos(\omega_c t + \theta)$?

{Recall $A\cos(\alpha) + B\sin(\alpha) = \left(A^2 + B^2\right)^{\frac{1}{2}}\cos(\alpha + \theta),\ \theta = -\tan^{-1}(\frac{B}{A}))$

$$E(t) = ((A + m(t))^2 + m_h^2(t))^{\frac{1}{2}} = ((A^2 + m^2(t)) + m_h^2(t) + 2Am(t))^{\frac{1}{2}}$$

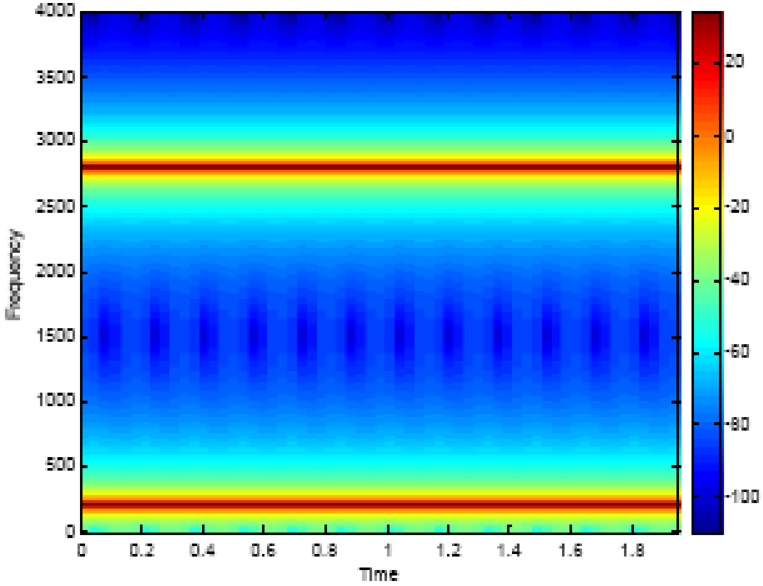
$$= A\left(1 + \frac{m^2(t)}{A^2} + \frac{m_h{}^2(t)}{A^2} + \frac{2m(t)}{A}\right)$$

$$\approx A + m(t) \qquad \text{for } A >> |m(t)|,\ A >> |m_h(t)|.$$

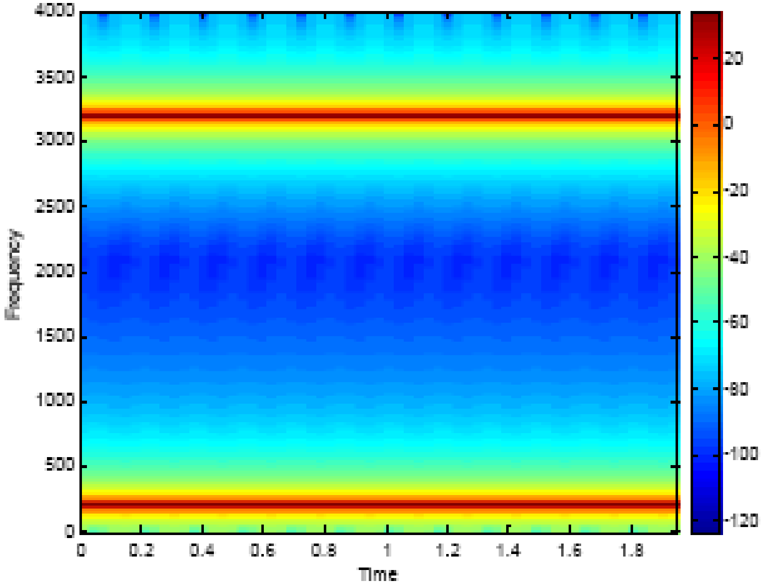
The efficiency of this scheme is very low since A has to be large.

Demodulation of SSB

Synchronous Demodulation can be used for SSB-SC signals



SSB-LSB



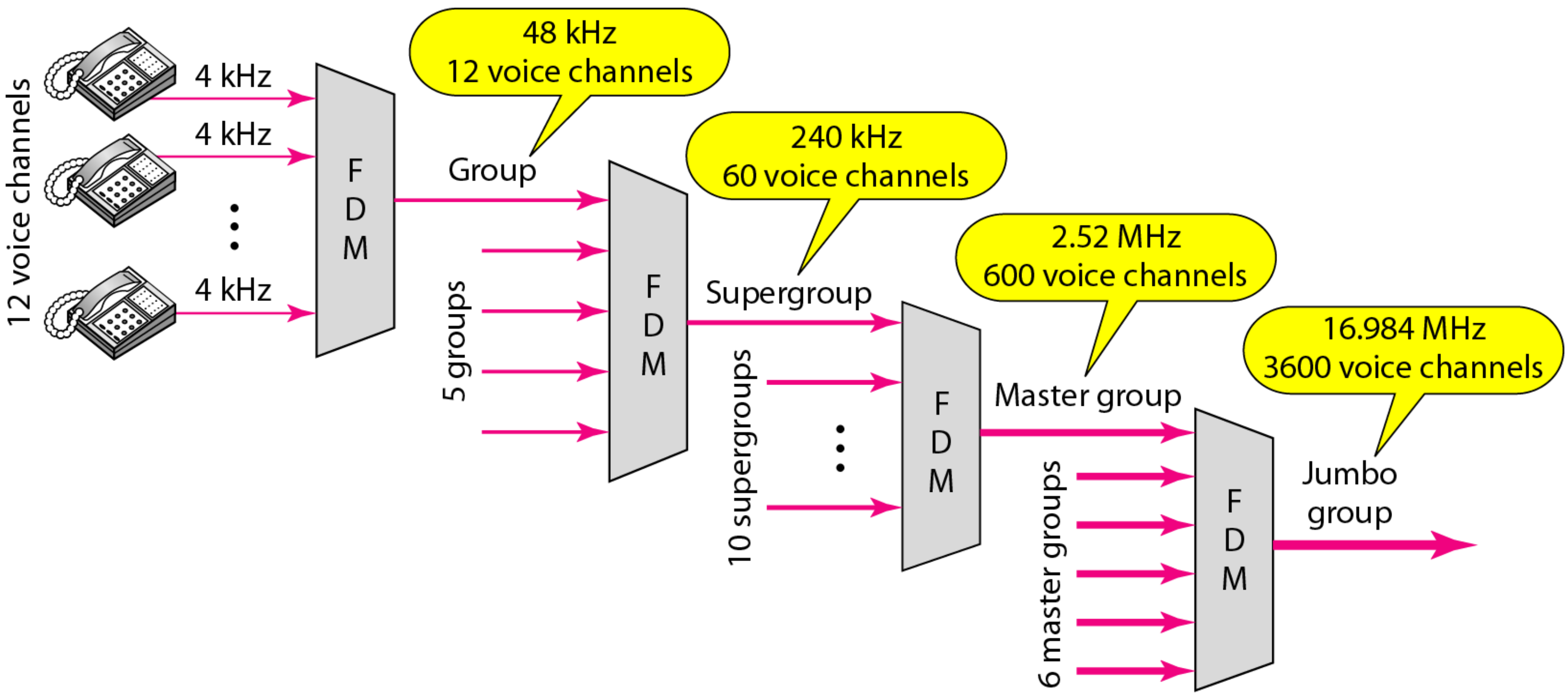
SSB-USB

SSB vs. AM

- Since the carrier is not transmitted, there is a reduction by 67% of the transmitted power (-4.7dBm). --In AM @100% modulation: 2/3 of the power is comprised of the carrier; with the remaining (1/3) power in both sidebands.
- Because in SSB, only one sideband is transmitted, there is a further reduction by 50% in transmitted power
- Finally, because only one sideband is received, the receiver's needed bandwidth is reduced by one half--thus effectively reducing the required power by the transmitter another 50%
- (-4.7dBm (+) -3dBm (+) -3dBm = -10.7dBm).
- Relative expensive receiver

Telephone Channel Multiplexing

Almost all long-haul telephone channels were multiplexed by FDM using SSB signals.

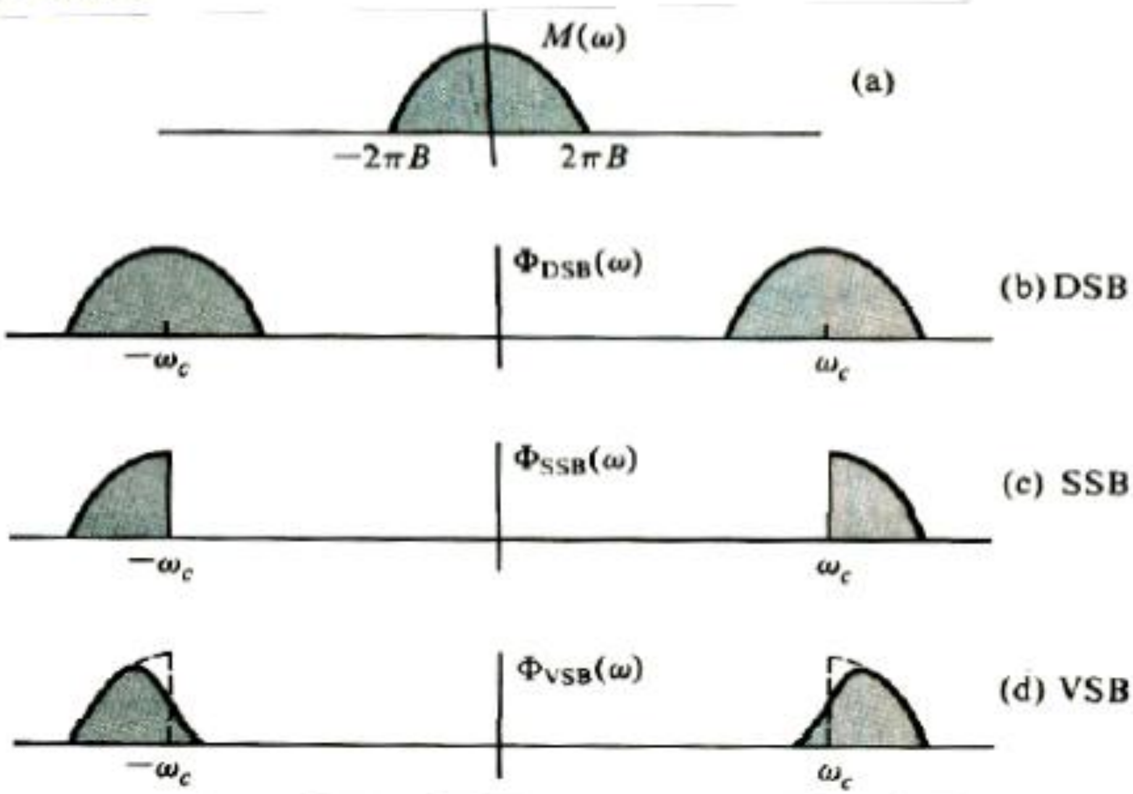


Amplitude Modulation: Vestigial Sideband (VSB)

- Vestigial Sideband (VSB)
- The generation of SSB signals is difficult
  - Selective Filtering method demands dc null in the modulating signal spectrum
  - A phase shifter required in the phase shift method is unrealizable
- A vestigial sideband system also known as asymmetric sideband system is a compromise between DSB and SSB

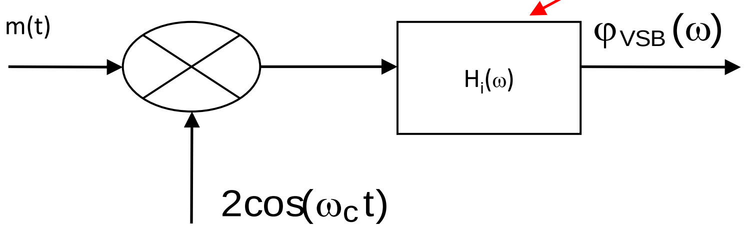
- VSB signals are relatively easy to generate and their bandwidth is typically 25 to 33% greater than that of SSB signals
- In VSB, instead of rejecting one sideband completely (as in SSB), a gradual cutoff of one sideband is accepted
- The baseband signal can be recovered exactly by a synchronous detector in conjunction with an appropriate equalizer filter $H_0(\omega)$ at the receiver output

VSB Spectra

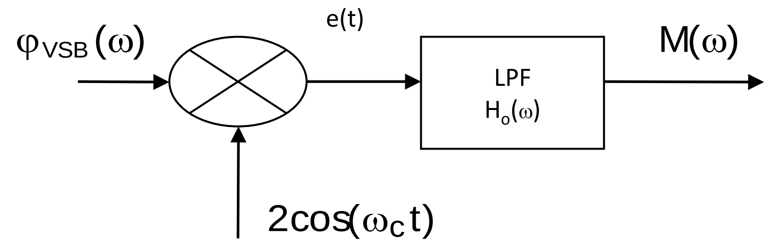


VSB Transceiver

VSB Shaping Filter



Transmitter



Receiver

$M(\omega)$ is bandlimited to $2\pi B$ rad/sec

$$\varphi_{VSB}(\omega) = [M(\omega-\omega_c) + M(\omega+\omega_c)]H_i(\omega)$$

$$E(\omega) = [\Phi_{VSB}(\omega-\omega_c) + \Phi_{VSB}(\omega+\omega_c)]$$

$$= [\underbrace{H_i(\omega-\omega_c)M(\omega-2\omega_c)}_{\text{High freq. term}} + H_i(\omega+\omega_c)M(\omega) + H_i(\omega-\omega_c)M(\omega) + \underbrace{H_i(\omega+\omega_c)M(\omega+2\omega_c)}_{\text{High freq. term}}]$$

$$\therefore \quad M(\omega) = E(\omega)H_o(\omega) = [H_i(\omega+\omega_c) + H_i(\omega-\omega_c)]M(\omega)H_o(\omega)$$

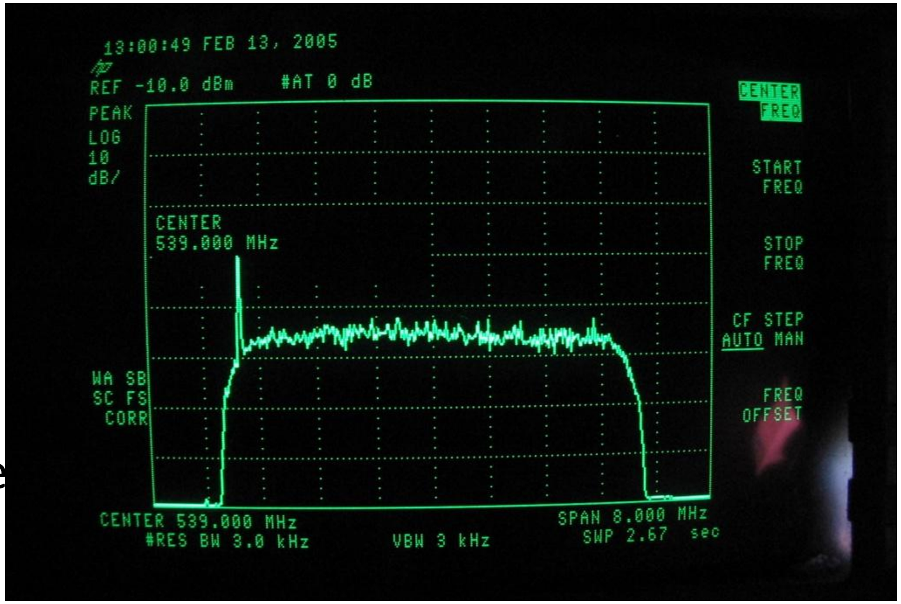
$$+ \underbrace{[H_i(\omega-\omega_c)M(\omega-2\omega_c) + H_i(\omega+\omega_c)M(\omega+2\omega_c)]H_o(\omega)}_{\text{Lowpass filter removes this.}}$$

Thus we should have $[H_i(\omega+\omega_c) + H_i(\omega-\omega_c)]H_o(\omega) = 1$ for $|\omega| \le 2\pi B$

OR $$H_o(\omega) = \frac{1}{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)}$$

Other Facts about VSB

- Envelope detection of VSB+C
- TV: Figure 4.24
- DSB, SSB and VSB
  - DSB bandwidth too high
  - SSB: baseband has low frequency component, receiver cost
  - Relax the filter and baseband requirement with modes increase in bandwidth



Comparison

- Common types & modulated signal

  1. AM: $s_{\text{AM}}(t) = A_c[1+m(t)]\cos(2\pi f_c t)$
  2. DSB: $s_{\text{DSB}}(t) = A_c m(t)\cos(2\pi f_c t)$
  3. QAM: $s_{\text{QAM}}(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$
  4. SSB: $s_{\text{SSB}}(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t)$
  5. VSB: $s_{\text{VSB}}(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \tilde{m}(t)\sin(2\pi f_c t)$

- Complex domain representation:

  $$s(t) = \text{Re}\{g(t)e^{j2\pi f_c t}\},$$

  complex envelop: $g(t) = ?$

- Bandwidth: $B_m \leq B \leq 2B_m$ (B_m: message bandwdith)

AM Broadcasting

- History
- Frequency
  - Long wave: 153-270kHz
  - Medium wave: 520-1,710kHz, AM radio
  - Short wave: 2,300-26,100kHz, long distance, SSB, VOA
- Limitation
  - Susceptibility to atmospheric interference
  - Lower-fidelity sound, news and talk radio
  - Better at night, ionosphere.

Questions?